

AN EXTENSION OF THE LAGUERRE POLYNOMIALS AND ASSOCIATED ORTHONORMAL FUNCTIONS

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Abstract

In this paper we present an extension of the generalized Laguerre polynomials in three variables and three indices. We also give recurrence relations related to this extension and the differential equations which these polynomials satisfy and define the associated orthonormal functions.

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Key Words and Phrases: generalized Laguerre polynomials, multivariable and multi-index polynomials, orthonormal functions

1. Introduction

The multindex and multivariable special functions are encountered in a variety of problems in applied mathematics and mathematical physics [5]. Dattoli et al [1,2] have treated generalized Bessel functions and Hermite polynomials. Galue et al. [7] have obtained multiplication theorems for generalized and double index Bessel functions. These multindex and multivariable polynomials and functions have applications in the theory of biorthogonal spaces and in the non-Hermitian realization of Weyl and angular momentum groups.

The ordinary Laguerre polynomials are defined in terms of the Hermite polynomials according to the relation [8]

$$L_p \left(\frac{x^2 + y^2}{2} \right) = \frac{(-1)^p}{2^p} \sum_{n=0}^p \frac{H_{2n}(x) H_{2p-2n}(y)}{n!(p-n)!} . \quad (1)$$

The Laguerre polynomials are another important class of orthogonal polynomials encountered in applications, especially in mathematical physics in problems